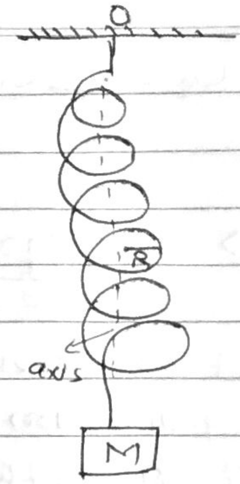
B.Sc - I, Paper - I, Group - A

FLAT SPIRAL SPRING :-



Let us consider a uniform wire of mass m, length L and radius r, is coiled on a cylinder of Radius R. In this way a helix is formed whose radius is R. Upper free end of helix is connected to a rigid support at O and the lower free end is loaded by a block of mass 'M'.

~~Due to the load Mg and external torque acts on the spring due to which it is~~

Due to load the helix is bended and twisted. If the boundary is very small than the plane of each turn of ~~of~~ helix ~~than the plane~~ remains almost horizontal and the axis of helix is perpendicular to it.

In this condition the helix is known as <u>flat spiral spring</u> and otherwise non flat spiral spring.

Due to load Mg and external torque acts on the spring due to which it is twisted. The internal torque developed in the spring opposes this torque. In equilibrium these two torques are equal in magnitude.

Let θ = angle of twist in equilibrium condition.

Internal restoring force = External torque due to the load.

i.e, $$\frac{\pi \eta r^4 \theta}{2l} = MgR \quad \text{——— (1)}$$

In this strained state the spring has strain potential energy given by

$$U_s = \frac{1}{2} C\theta^2 \qquad \left\{ C = \text{torsional rigidity} = \frac{\pi\eta r^4}{2l} \right.$$

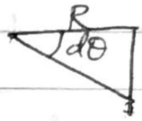
$$\Rightarrow U_s = \frac{\pi\eta r^4\theta^2}{4l} \quad \text{——— (2)}$$

let us consider the load is slightly depressed by 'x' from equilibrium position.

If '$d\theta$' is the change in the twist θ.

increase in strain potential energy of the spring.

$$\Delta U_s = \frac{\pi\eta r^4 (d\theta)^2}{4l}$$



$$= \frac{\pi\eta r^4}{4l} \cdot \frac{x^2}{R^2} \quad \text{——— (3)}$$

Decrease in gravitational potential energy of the ~~lod~~ load $= Mgx$ ——— (4)

The c.g of the spring is lowered by $\frac{x}{2}$.

So, decrease in gravitational potential energy of the spring $= mg\frac{x}{2}$ ——— (5)

Hence, Total potential energy of the system (spring + total) when the load is at displaced position

$$U = \frac{\pi\eta r^4\theta^2}{4l} + \frac{\pi\eta r^4 x^2}{4lR^2} - Mgx - Mg\frac{x}{2} \quad \text{——— (6)}$$

Displacement of the load $= x$

$\therefore$ Instantaneous velocity of the load $= \frac{dx}{dt} = v$

$\therefore$ Kinetic energy of the load $K_L = \frac{1}{2}M\left(\frac{dx}{dt}\right)^2$ —— (7)

To find KE of the spring :-

Let us consider an element of length dz at a distance z from fixed and of the spring.

Mass of the element $= dM = \frac{m}{L}dz$

and velocity of the element $= \frac{zv}{L} = \frac{z}{L}\frac{dx}{dt}$

K.E of the element

$$dK_s = \frac{1}{2}\left(\frac{m}{L}dz\right)\frac{z^2}{L^2}\left(\frac{dx}{dt}\right)^2$$

K.E of the spring $-\; K_s = \frac{m}{2L^3}\left(\frac{dx}{dt}\right)^2\int_0^z z^2 dz$

$$= \frac{m}{2L^3}\left(\frac{dx}{dt}\right)^2\frac{L^3}{3}$$

$$= \frac{m}{6}\left(\frac{dx}{dt}\right)^2 \quad \text{—— (8)}$$

So total kinetic energy Conservation the total energy

$E = U + K = \text{Constant}$.

$$= \frac{\pi\eta r^4\theta^2}{4l} + \frac{\pi\eta r^4 x^2}{4lR^2} - mgx - mg\frac{x}{2} + \frac{1}{2}M\left(\frac{dx}{dt}\right)^2 + \frac{1}{6}m\left(\frac{dx}{dt}\right)^2 = \text{Constant}$$

$$\Rightarrow \frac{1}{2}\left(M + \frac{m}{3}\right)\left(\frac{dx}{dt}\right)^2 + \frac{\pi\eta r^4 x^2}{4lR^2} - g\left(M + \frac{m}{2}\right)x = \text{Constant}$$

$$\left[\text{as, } \frac{\pi\eta r^4\theta^2}{4l} = \text{Constant}\right]$$

Differentiating w.r.to "t".

$$\frac{1}{2}\left(M+\frac{m}{3}\right)\times\left(2\frac{dx}{dt}\cdot\frac{d^2x}{dt^2}\right)+\frac{\pi\eta r^4}{4lR^2}\times\left(2x\frac{dx}{dt}\right)-g\left(M+\frac{m}{2}\right)\frac{dx}{dt}=0.$$

$$\Rightarrow\left(M+\frac{m}{3}\right)\frac{d^2x}{dt^2}+\frac{\pi\eta r^4}{2lR^2}x-g\left(M+\frac{m}{2}\right)=0$$

$$\Rightarrow\left(M+\frac{m}{3}\right)\frac{d^2x}{dt^2}+\frac{\pi\eta r^4}{2lR^2}\left[x-\frac{g\left(M+\frac{m}{2}\right)\cdot 2lR^2}{\pi\eta r^4}\right]=0$$

let, $x-\frac{g\left(M+\frac{m}{2}\right)\cdot 2lR^2}{\pi\eta r^4}=x'$

$$\therefore \frac{d^2x}{dt^2}=\frac{d^2x'}{dt^2}$$

$$\left(M+\frac{m}{3}\right)\frac{d^2x'}{dt^2}+\frac{\pi\eta r^4}{2lR^2}x'=0$$

$$\Rightarrow \frac{d^2x'}{dt^2}+\frac{\pi\eta r^4}{2lR^2\left(M+\frac{m}{3}\right)}\cdot x'=0 \quad \text{——— (9)}$$

This is the eqn. of S.H.M.
So, the system ossilate simple Harmonically.

Angular frequency $\omega=\sqrt{\frac{\pi\eta r^4}{2lR^2\left(M+\frac{m}{3}\right)}}$

Time period $T=\frac{2\pi}{\omega}=2\pi\sqrt{\frac{2lR^2\left(M+\frac{m}{3}\right)}{\pi\eta r^4}}$ ——— (10)

Thus, the load performs S.H.M after a small displacement from its equilibrium position. By measuring the time period of oscillation.